Common Errors With Logarithm Properties

The following are errors students often make when working with logarithm properties. Since I can't stand to write incorrect statements, the common errors are stated as inequalities.

Error One

$$\log_a(M \pm N) \neq \log_a M \pm \log_a N$$

The logarithm function does not respect sums or differences. There is no property for the logarithm of a sum or a difference.

Error Two

$$\log_a(M + N) \neq (\log_a M)(\log_a N)$$

This is mixing up what the product rule for logarithms actually says. As stated above, there is no property for the logarithm of a sum. There is also no property for the product of two logarithms.

Error Three

$$\log_a(M - N) \neq \frac{\log_a M}{\log_a N}$$

This is mixing up what the quotient rule for logarithms actually says. As stated above, there is no property for the logarithm of a difference. There is also no property for the quotient of two logarithms (except the change of base formula).

Error Four

$$\log_a MN \neq (\log_a M)(\log_a N) \quad \text{and} \quad \log_a \frac{M}{N} \neq \frac{\log_a M}{\log_a N}$$

The logarithm function does not respect product and quotients in this manner. As stated above, there are no properties for the right hand sides of the expressions above (except for the change of base formula for the quotient).

Error Five

$$\log_a M + \log_a N \neq (\log_a M)(\log_a N)$$

This is a misstatement of the product rule. The product on the right hand side should be the product of the arguments, inside the parentheses.

Error Six

$$\log_a M - \log_a N \neq \frac{\log_a M}{\log_a N}$$

This is a misstatement of the quotient rule. The quotient on the right hand side should be the quotient of the arguments, inside the parentheses.

Error Seven

$$r \log_a M \neq (\log_a M)^r$$

This is a misstatement of the power rule. The power on the right hand side should be a power on the argument inside the parentheses.